GCSE problem solving; its importance, and some practice

Gary Thomson

September 2022

Solving problems is a practical art, like swimming, or skiing, or playing the piano ...if you wish to learn swimming you have to go into the water, if you wish to become a problem-solver you have to solve problems. George Polya

The table below is from page 43 of the Pearson ICCSE Mathematics (Specification A) (9-1)

Relationship of problem-solving and mathematical reasoning skills to tier.

	Problem solving	Mathematical reasoning
Foundation (1F and 2F)	25%	15%
Higher (1H and 2H)	30%	20%

If I am reading it correctly it says that at Higher tier 50% of the course content (or marks in the exam – check with your teacher) are for problem solving and mathematical reasoning. Therefore in order to succeed with a 9 you have take seriously the task of learning how to solve mathematical problems. Unfortunately it is not something that can be compelled, it can however be encouraged and supported.

One big difference in shifting the emphasis to problem solving is that the focus is taken away from getting the correct answer to that of finding a fruitful approach which will lead to a correct answer. Finding this approach may involve several missteps, and so careful checking and perseverance are skills that go hand in hand with it. Also involved is a focus on *understanding* (which is where maths should be focused), and on being able to apply one's knowledge. It is for this reason that learning to solve problems mathematically is a skill that is valued by employers.

1 STRATEGIES

(these map to the questions below as indicated)

2.1 Read the question, and think
2.2 Think, guess, and check
2.3 Experiment, and convince yourself of your findings
2.4 Apply your knowledge, and check
2.5 Apply your knowledge
3.1 and 4.1 Apply your knowledge, take your time (don't expect it to be clear immediately) you have to think

2.3 is a question where you may not be sure, at this stage, of how to proceed in order to convince yourself of your findings.

3.1 and 4.1 may involve areas af IGCSE maths that you've not covered yet.

Problem solving means becoming comfortable with *not* knowing what to do, and that is a difficult thing for many people. Nonetheless we can become more at ease with not knowing, allowing us to approach problems with greater openness and creativity, where we're less bothered about getting the right answer and more concerned with understanding the problem and using what we already know to solve it. In doing this our confidence grows and accelerates our learning of mathematics.

It's also worth repeating that to learn problem solving will involve you in making many mistakes, as this is the principal means of learning in becoming a problem solver. So do not be put off by your errors be they small or great, keep going, keep learning.

2 Easy

2.1 The UK Mathematical Challenge

A transport company's vans carry a maximum load of 12 tonnes. A firm needs to deliver 24 crates each weighing 5 tonnes. How many van loads will be needed to do this?

2.2 Junior Olympiad 1995 Section A

Evaluate $123123 \div 1001$

2.3 Junior Olympiad 1995 Section B

Take any two-digit number. Subtract the sum of its digits. Then divide the answer by 9. What do you find? Explain.
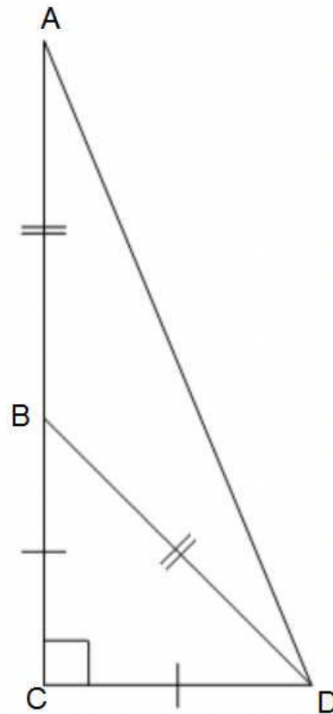
2.4 Elastic numbers: Bronze

What is $1/2$ of $1/3$ of $1/4$ of 168?

2.5 GCSE Maths Challenge Tough problems with solutions, Mark Ritchings – Kindle edition

If the density of gold is 19.3 tonnes/m^3 and the price of gold is £47.19/g, find the value of a cube of gold the edges of which are 5cm in length. You may use a calculator.

3 A bit harder

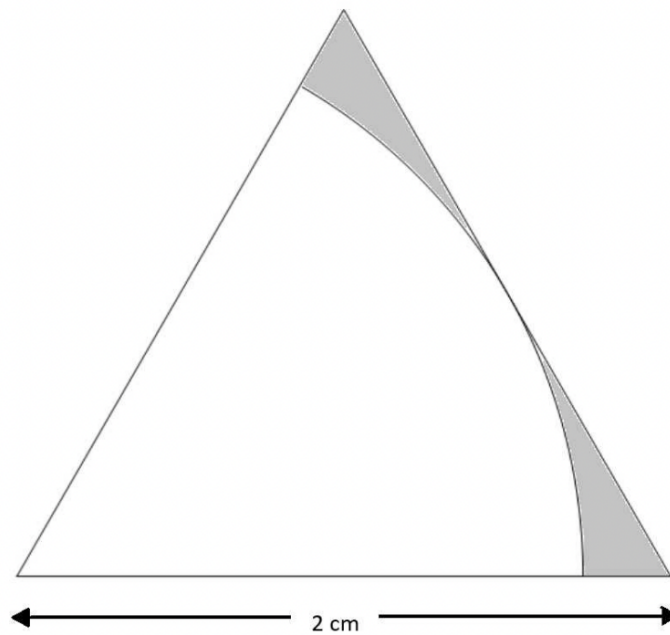## 3.1 GCSE Maths Challenge Tough problems with solutions, Mark Ritchings – Kindle edition

$BC = 1$.
Find the exact value of $\tan 22.5°$.

4 Hard

4.1 GCSE Maths Challenge Tough problems with solutions, Mark Ritchings – Kindle edition

The diagram shows an equilateral triangle and a sector of a circle the centre of which is one of the vertices of the triangle. Find the area shaded in the diagram, giving your answer in its exact form.

Bibliography

The Ultimate Mathematical Challenge The UK Mathematics Trust Harper Collins, More Mathematical Challenges Tony Gardiner Cambridge, Elastitc Numbers Daniel Griller Rational Falcon, GCSE Maths Challenge Tough problems with solutions, Mark Ritchings