Optimisation: maxima and minima

[Please note that HHM below refers to Heinemann Higher Mathematics]

The idea here is to use calculus to solve problems where the key to the solution is seeing that a single variable can be varied, under some constraint, in such a way as to produce either a maximum value, or a minimum value, in some quantity in which we are interested.

These maxima or minima will be at **stationary points** on the graph of the function which describes the situation we are being asked to investigate. Therefore we need to find *the values of x for which the derived function equals zero*.

So in Example 24 (p111 of HHM) we are told:

- Breadth = x
- Total length of gold leaf = 16cm

and are asked to:

- Find the dimensions of the rectangle which give the **maximum** area
- Calculate this area

So

- x is the variable
- 16 is the constraint

One other thing to note in this example is that the function which describes the situation is a quadratic; take note of the sign of the co-efficient of the x^2 [*x^2 is read as "x squared"*] term. In this example

A(x) = 8x – x^2

and the co-efficient of the x^2 term is -1. This corresponds to a "sad" parabola and therefore a **maximum** value at the turning point. That matches what we are being asked to find (*the dimensions which give the **maximum** area*).

When the function is a quadratic this can be a good check that your function is along the right lines.

- **+ve** coefficient of x^2 means "happy" parabola and so **minimum** turning point
- **-ve** coefficient of x^2 means "sad" parabola and so **maximum** turning point

Ex 6Q

Q1.

- x, length of the base of rectangle, is the variable
- perimeter = 36cm is the constraint
- asked to find dimensions for, and to calculate, the maximum area
- the expression for the area involves a –ve coefficient of x^2 and so corresponds to a maximum

Q2.

- x, breadth of window, is the variable
- perimeter = 25m is the constraint
- asked to find dimensions for, and to calculate, the maximum area
- the expression for the area involves a –ve coefficient of x^2 and so corresponds to a maximum

Q3.

- x, the offset from the corner, is the variable
- 60 is the constraint
- the expression for the area involves a +ve coefficient of x^2 and so corresponds to a minimum
- find the minimum area

Here the thing to notice is that as x varies so do the other lengths in the corners of rectangle PQRS. If we imagine making x smaller and smaller the parallelogram ABCD would fit over the rectangle PQRS, and so its area would be 60. So 60 is the maximum area. We are asked to find a minimum area.

One way to see what this area must be is to see that the area of the parallelogram is the area of the rectangle minus the area of the four right angled triangles in the corners of the rectangle.

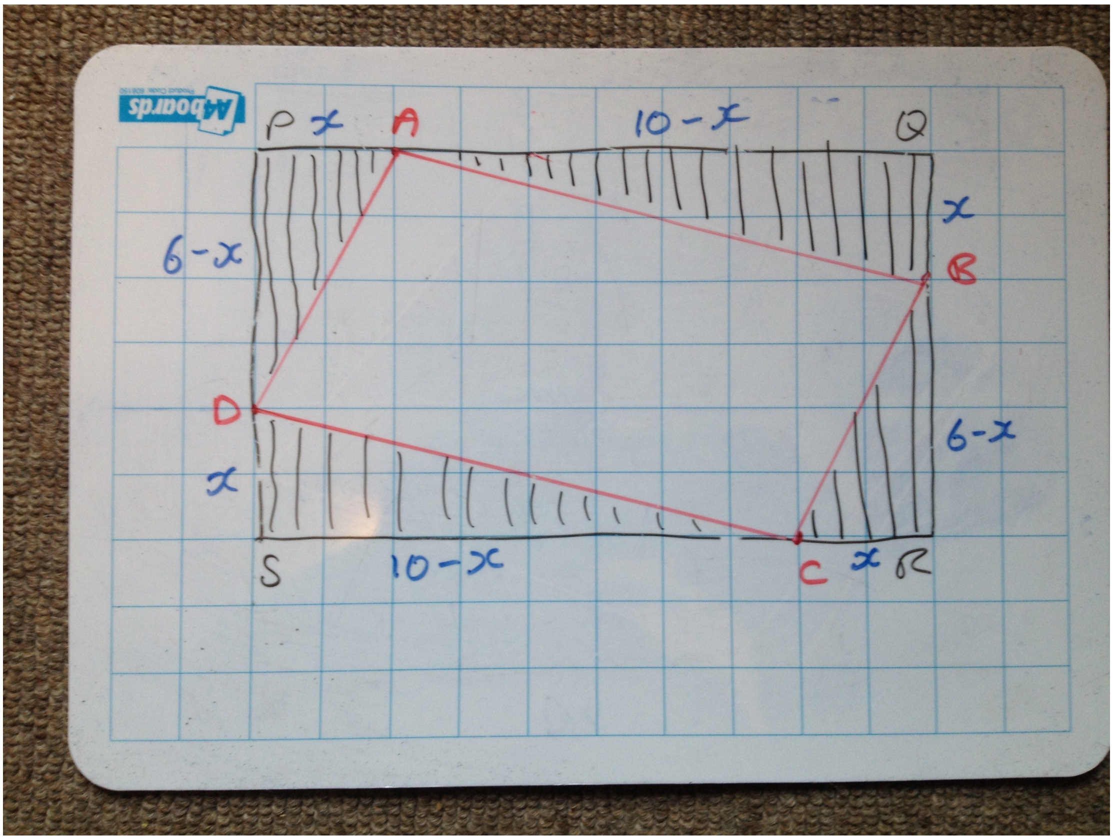

Figure 1 my attempt to show that the area of the parallelogram is the area of the rectangle minus the 4 RAT

When doing problems like this one, where the function is given to you, please keep your eye on the function as you work with the information in the question.

Q4.

- x, breadth of enclosure, is the variable
- 8m^2, area of the enclosure, is the constraint
- find the dimensions of enclosure which will give the minimum perimeter

In this instance the function which gives the perimeter in terms of x does not involve an x^2 term so we cannot use our rule which helps determine if we should get a maximum or a minimum.